Operators
and
Matrices
Volume 9, Number 1 (2015), 189–201
doi:10.7153/oam-09-11

ORTHONORMAL JORDAN BASES IN FINITE DIMENSIONAL HILBERT SPACES

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Abstract. Necessary and sufficient conditions are presented for a linear operator in a finite dimensional complex or real Hilbert space to have a Jordan form in an orthonormal basis. Further, necessary conditions are given in terms of the self-commutator operator.

Mathematics subject classification (2010): 15A21, 15A04, 15B57.
Keywords and phrases: orthonormal Jordan basis, self-commutator of operator or matrix, multiset, multiplicity, power partial isometry, Jordan-Dunford decomposition.

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