

## HIGHER RANK NUMERICAL HULLS OF MATRICES AND MATRIX POLYNOMIALS

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*Abstract.* In this paper, some properties of the higher rank numerical hulls, as a generalization of higher rank numerical ranges and polynomial numerical hulls, of matrices are investigated. In particular, the higher rank numerical hulls of Pauli matrices are characterized. Moreover, the notion of higher rank numerical hulls of matrix polynomials is introduced, and some algebraic properties of this notion are investigated. The higher rank numerical hulls of the basic  $A$ -factor block circulant matrix, which is the block companion matrix of the matrix polynomial  $Q(\lambda) = \lambda^s I_n - A$ , are also studied.

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