

HOCHSCHILD COHOMOLOGY OF TYPE II_1 VON NEUMANN ALGEBRAS WITH PROPERTY Γ

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Abstract. In this paper, Property Γ for a type II_1 von Neumann algebra is introduced as a generalization of Murray and von Neumann's Property Γ for a type II_1 factor. The main result of this paper is that if a type II_1 von Neumann algebra \mathcal{M} with separable predual has Property Γ , then the continuous Hochschild cohomology group $H^k(\mathcal{M}, \mathcal{M})$ vanishes for every $k \geq 2$. This gives a generalization of an earlier result in [4].

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