

ON THE DISTANCE TO SINGULARITY VIA LOW RANK PERTURBATIONS

CHRISTIAN MEHL, VOLKER MEHRMANN AND MICHAŁ WOJTYŁAK

Abstract. For regular matrix pencils the distance in norm to the nearest singular pencil under low rank perturbation is studied. Characterizations of this distance are derived via the Weyl function of the perturbation. Special attention is paid to the Hermitian pencil case. Estimates for the distance of a given pencil to the set of singular pencils are obtained.

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