

C^* -ALGEBRAS OF BERGMAN TYPE OPERATORS WITH CONTINUOUS COEFFICIENTS ON POLYGONAL DOMAINS

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Abstract. Given $\alpha \in (0, 2]$, the C^* -algebra $\mathfrak{A}_{\mathbb{K}\alpha}$ generated by the identity operator and by the Bergman and anti-Bergman projections acting on the Lebesgue space $L^2(\mathbb{K}_\alpha)$ over the open sector

$$\mathbb{K}_\alpha = \{z = re^{i\theta} : r > 0, \theta \in (0, \pi\alpha)\}$$

is studied. Then, for any bounded polygonal domain U , the C^* -algebra \mathfrak{B}_U generated by the operators of multiplication by continuous functions on the closure \overline{U} of U and by the Bergman and anti-Bergman projections acting on the Lebesgue space $L^2(U)$ is investigated. Symbol calculi for the C^* -algebras $\mathfrak{A}_{\mathbb{K}\alpha}$ and \mathfrak{B}_U are constructed and an invertibility criterion for operators $A \in \mathfrak{A}_{\mathbb{K}\alpha}$ and a Fredholm criterion for the operators $B \in \mathfrak{B}_U$ in terms of their symbols are established.

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