

## MODEL SPACES AND TOEPLITZ KERNELS IN REFLEXIVE HARDY SPACE

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*Abstract.* This paper considers model spaces in an  $H_p$  setting. The existence of unbounded functions and the characterisation of maximal functions in a model space are studied, and decomposition results for Toeplitz kernels, in terms of model spaces, are established.

*Mathematics subject classification (2010):* 47B35, 30H10.

*Keywords and phrases:* Model space, Toeplitz operator, Hardy space, near invariance.

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