

## CLOSED IDEALS IN $\mathcal{L}(X)$ AND $\mathcal{L}(X^*)$ WHEN $X$ CONTAINS CERTAIN COPIES OF $\ell_p$ AND $c_0$

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*Abstract.* Suppose  $X$  is a real or complexified Banach space containing a complemented copy of  $\ell_p$ ,  $p \in (1, 2)$ , and a copy (not necessarily complemented) of either  $\ell_q$ ,  $q \in (p, \infty)$ , or  $c_0$ . Then  $\mathcal{L}(X)$  and  $\mathcal{L}(X^*)$  each admit continuum many closed ideals. If in addition  $q \geq p'$ ,  $\frac{1}{p} + \frac{1}{p'} = 1$ , then the closed ideals of  $\mathcal{L}(X)$  and  $\mathcal{L}(X^*)$  each fail to be linearly ordered. We obtain additional results in the special cases of  $\mathcal{L}(\ell_1 \oplus \ell_q)$  and  $\mathcal{L}(\ell_p \oplus c_0)$ ,  $1 < p < 2 < q < \infty$ .

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### REFERENCES

- [1] Y. A. ABRAMOVICH AND C. D. ALIPRANTIS, *An Invitation to Operator Theory*, Graduate Studies in Mathematics, Vol. **50**, The American Mathematical Society (2002), ISBN 0-8218-2146-6.
- [2] FERNANDO ALBIAC AND NIGEL J. KALTON, *Topics in Banach Space Theory*, Graduate Texts in Mathematics, Springer Inc. (2006), ISBN 978-0387-28141-4.
- [3] JOSEPH DIESTEL, *Sequences and Series in Banach Spaces*, Graduate Texts in Mathematics **92** (1984), ISBN 0-387-90859-5.
- [4] JOE DIESTEL, HANS JARCHOW, AND ANDREW TONGE, *Absolutely Summing Operators*, Cambridge studies in advanced mathematics **43** (1995), ISBN 0-521-43168-9.
- [5] KARL LINDBERG, *On subspaces of Orlicz sequence spaces*, Studia Mathematica (1973) Vol. **45**, No. 2, pp. 119–146.
- [6] JORAM LINDENSTRAUSS AND LIOR TZAFRIRI, *On Orlicz sequence spaces II*, Israel Journal of Mathematics (December 1972) Vol. **11**, No. 4, pp. 355–379.
- [7] JORAM LINDENSTRAUSS AND LIOR TZAFRIRI, *Classical Banach Spaces I: Sequence Spaces*, (1977), ISBN 3-540-08072-4.
- [8] ROBERT E. MEGGINSON, *An Introduction to Banach Space Theory*, Graduate Texts in Mathematics 183, Springer-Verlag New York, Inc. (1998), ISBN 0-387-98431-3
- [9] V. D. MILMAN, *Spectrum of bounded continuous functions specified on a unit sphere in Banach space* (English translation from Russian), Functional Analysis and Its Applications (April-June, 1969), Vol. **3**, No. 2, pp. 137–146.
- [10] V. D. MILMAN, *Operators of class  $C_0$  and  $C_0^*$*  (Russian), Teor. Funkciĭ Funkcional. Anal. i Priložen. (1970), Vol. **10**, pp. 15–26.
- [11] ALBRECHT PIETSCH, *Operator Ideals*, Mathematische Monographien, Vol. **16**, Deutscher Verlag der Wissenschaften (1978).
- [12] GILLES PISIER, *Holomorphic semi-groups and the geometry of Banach spaces*, Annals of Mathematics Second Series, (March 1982), Vol. **115**, No. 2, pp. 375–392.
- [13] A. PLICHKO, *Superstrictly singular and superstrictly cosingular operators*, North-Holland Mathematics Studies (July 2004), Vol. **197**, Functional analysis and its applications, pp. 239–255, ISBN 0-444-51373-6.
- [14] HASKELL P. ROSENTHAL, *On the subspaces of  $L_p$  ( $p > 2$ ) spanned by sequences of independent random variables*, Israel J. Math. (1970), Vol. **8**, pp. 273–303.
- [15] THOMAS SCHLUMPRECHT, *On the closed subideals of  $L(\ell_p \oplus \ell_q)$* , Operators and Matrices (2012), Vol. **6**, 311–326.

- [16] B. SARI, THOMAS SCHLUMPRECHT, NICOLE TOMCZAK-JAEGERMANN, VLADIMIR TROITSKY, *On norm closed ideals in  $L(\ell_p, \ell_q)$* , *Studia Math.* (2007), Vol. **179**, No. 3, pp. 239–262.
- [17] THOMAS SCHLUMPRECHT, ANDRÁS ZSÁK, *The algebra of bounded linear operators on  $\ell_p \oplus \ell_q$  has infinitely many closed ideals*, (2014, preprint).
- [18] JOSEPH Y. T. WOO, *On a class of universal modular sequence spaces*, *Israel J. Math.* (1975), Vol. **20**, No. 3–4, pp. 193–215.