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VARIATIONAL PRINCIPLES FOR SELF–ADJOINT OPERATOR FUNCTIONS ARISING FROM SECOND–ORDER SYSTEMS

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Abstract. Variational principles are proved for self-adjoint operator functions arising from variational evolution equations of the form

$$\langle \ddot{z}(t), y\rangle + \mathfrak{d}[\dot{z}(t), y] + \mathfrak{a}_0[z(t), y] = 0.$$

Here $\mathfrak{a}_0$ and $\mathfrak{d}$ are densely defined, symmetric and positive sesquilinear forms on a Hilbert space H. We associate with the variational evolution equation an equivalent Cauchy problem corresponding to a block operator matrix $\mathcal{A}$, the forms

$$\mathfrak{t}(\lambda)[x,y] := \lambda^2\langle x, y\rangle + \lambda\,\mathfrak{d}[x,y] + \mathfrak{a}_0[x,y],$$

where $\lambda \in \mathbb{C}$ and x, y are in the domain of the form $\mathfrak{a}_0$, and a corresponding operator family $T(\lambda)$. Using form methods we define a generalized Rayleigh functional and characterize the eigenvalues above the essential spectrum of $\mathcal{A}$ by a min-max and a max-min variational principle. The obtained results are illustrated with a damped beam equation.

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