

SPECTRAL PROPERTIES OF k -QUASI- $*$ - n -PARANORMAL OPERATORS

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Abstract. For positive integers n and k , an operator T is said to be k -quasi- $*$ - n -paranormal if $\| |T^{1+n+k}x| \|_{\frac{1}{1+n}} \| |T^kx| \|_{\frac{n}{1+n}} \geq \| |T^*T^kx| \|$ for all $x \in H$, which is a generalization of $*$ -paranormal operator. In this paper, we prove that the spectrum is continuous on the class of all k -quasi- $*$ - n -paranormal operators. Let λ be an isolated point of $\sigma(T)$ and E be the Riesz idempotent with respect to λ . We also prove that (1) if $\lambda \neq 0$, then E is self-adjoint and $R(E) = N(T - \lambda) = N(T - \lambda)^*$. (2) if $\lambda = 0$, then $R(E) = N(T^{k+1})$.

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