

NON-SELF-ADJOINT FOURTH-ORDER DISSIPATIVE OPERATORS AND THE COMPLETENESS OF THEIR EIGENFUNCTIONS

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Abstract. A class of non-self-adjoint fourth order differential operators with general separated boundary conditions in Weyl's limit circle case is studied. The dissipation property of the considered operators in $L^2(a, b)$ is proven by analysis and by using the characteristic determinant, the completeness of the system of eigenfunctions and associated functions of these dissipative operators also be proven.

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