

THE TRACE AS AN AVERAGE OVER THE UNIT SPHERE OF A NORMED SPACE WITH A 1-SYMMETRIC BASIS

TOMASZ KANIA AND KENT E. MORRISON

Abstract. We generalise the formula expressing the matrix trace of a given square matrix as the integral of the numerical values of A over the Euclidean sphere to the unit spheres of finite-dimensional normed spaces that have a 1-symmetric basis. Our result is new even in the case of ℓ_p -norms in \mathbb{R}^N for $p \neq 2$.

Mathematics subject classification (2010): 15A60, 47A12.

Keywords and phrases: matrix trace; numerical range; hypersurface measure; hyperoctahedral group.

REFERENCES

- [1] B. BEAUZAMY, *Introduction to Banach spaces and their geometry*, Mathematics Studies, **68**, North-Holland, Amsterdam, 1985.
- [2] S. GALLOT, D. HULIN AND J. LAFONTAINE, *Riemannian geometry*, Universitext, Springer-Verlag, Berlin, Heidelberg, New York, 1987.
- [3] T. KANIA, *A short proof of the fact that the matrix trace is the expectation of the numerical values*, Amer. Math. Monthly Vol. **122**, 8 (2015), 782–783.
- [4] E. M. STEIN, *Singular integrals and differentiability properties of functions*, Mathematical Series No. **30**, Princeton University Press, Princeton, NJ, 1970.