

INVERSE PROBLEMS FOR SELF-ADJOINT DIRAC SYSTEMS: EXPLICIT SOLUTIONS AND STABILITY OF THE PROCEDURE

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Abstract. A procedure to recover explicitly self-adjoint matrix Dirac systems on the semi-axis (with both discrete and continuous components of spectrum) from rational Weyl functions is considered. Its stability is proved. GBDT version of Bäcklund-Darboux transformation and various important results on Riccati equations are used for this purpose.

Mathematics subject classification (2010): 15A24, 34A55, 34B20, 34D20, 93B20.

Keywords and phrases: Inverse problem, stability, Dirac system, Weyl function, minimal realization, explicit solution, Riccati equation.

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