

THE EFFECT OF PERTURBATIONS OF OPERATOR-VALUED FRAME SEQUENCES AND FUSION FRAMES ON THEIR DUALS

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Abstract. Fusion frames, and, more generally, operator-valued frame sequences are generalizations of classical frames, which are today a standard notion when redundant, yet stable sequences are required. However, the question of stability of duals with respect to perturbations has not been satisfactorily answered. In this paper, we quantitatively measure this stability by considering the associated deviations of the canonical and alternate dual sequences from the original ones. It is proven that operator-valued frame sequences are indeed stable in this sense. Along the way, we also generalize existing definitions for fusion frame duals to the infinite-dimensional situation and analyze how they perform with respect to a list of desiderata which, to our minds, a fusion frame dual should satisfy. Finally, we prove a similar stability result as above for fusion frames and their canonical duals.

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