

## AN OBSERVATION ABOUT NORMALOID OPERATORS

JOR-TING CHAN AND KONG CHAN

*Abstract.* Let  $H$  be a complex Hilbert space and  $B(H)$  the Banach space of all bounded linear operators on  $H$ . For any  $A \in B(H)$ , let  $w(A)$  denote the numerical radius of  $A$ . Then  $A$  is normaloid if  $w(A) = \|A\|$ . In this note, we show that  $A$  is normaloid if there is a sequence of unit vectors  $(x_n)$  such that  $\lim_{n \rightarrow \infty} \|Ax_n\| = \|A\|$  and  $\lim_{n \rightarrow \infty} |\langle Ax_n, x_n \rangle| = w(A)$  simultaneously. The result is then used to study the Davis-Wielandt radius.

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