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INVERSE AND MOORE–PENROSE INVERSE OF TOEPLITZ MATRICES WITH CLASSICAL HORADAM NUMBERS

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Abstract. For integers s,k with $s \leqslant 0$ and $k \geqslant 0$, we define a class of lower triangular Toeplitz matrices $\mathscr{U}_n^{(s,k)}$ of type (s,k), whose non-zero entries are the classical Horadam numbers $U_i^{(a,b)}$. In this paper, we derive a convolution formula containing the Horadam numbers. Using this formula, we obtain several combinatorial identities involving the Horadam numbers and the generalized Fibonacci numbers. In addition, we derive the inverse of the lower triangular Toeplitz matrix $\mathscr{U}_n^{(0,k)}$ and the Moore-Penrose inverse of the strictly lower triangular Toeplitz matrix $\mathscr{U}_n^{(s,k)} (s<0)$ by utilizing only the Horadam numbers.

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