

SHAPIRO'S UNCERTAINTY PRINCIPLE RELATED TO THE WINDOWED FOURIER TRANSFORM ASSOCIATED WITH THE RIEMANN-LIOUVILLE OPERATOR

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Abstract. Quantitative Shapiro's dispersion uncertainty principle and umbrella theorem are proved for the windowed Fourier transform associated with the Riemann-Liouville operator.

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