

## SOME PROPERTIES OF G-FRAMES FOR HILBERT SPACE OPERATORS

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*Abstract.* In this paper, we present some new characterizations of g-frames for a bounded operator. We discuss the g-frame for an operator  $K$  which has closed range and give a necessary and sufficient conditions for a family of bounded operators to be a  $K$ -g-frame. We also give a characterization of the dual for a  $K$ -g-frame. Moreover, We use quotient operators to characterize  $K$ -g-frames and find that the results on  $K$ -g-frames can be proved by theory of quotient operators.

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