

## DISTANCE ESTIMATES, NORM OF HANKEL OPERATORS AND RELATED QUESTIONS

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*Abstract.* We consider Berezin symbols and Hankel operators on the Hardy space  $H^2(\mathbb{D})$  over the unit disc  $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$  and give their some applications. Namely, we estimate in terms of Hankel operators and Berezin symbols the distances from a given operator to the algebra of all analytic Toeplitz operators and to the set of all Toeplitz operators on  $H^2(\mathbb{D})$ . We use Hankel operator also to prove some lower estimate for the so-called Berezin number of bounded linear operators on  $H^2$ . Some other related questions are also discussed.

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