

SCALAR APPROXIMANTS OF QUADRATIC OPERATORS WITH APPLICATIONS

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Abstract. Among other results, we find the best scalar approximant of a quadratic operator with respect to the numerical radius and the operator norm. We use these results to give estimates for the numerical radii of products and commutators of quadratic operators.

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