

DERIVATIONS ON TERNARY RINGS OF OPERATORS

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Abstract. To each projection p in a C^* -algebra A we associate a family of derivations on A , called p -derivations, and relate them to the space of triple derivations on $pA(1-p)$. We then show that every derivation on a ternary ring of operators is spatial and we investigate whether every such derivation on a weakly closed ternary ring of operators is inner.

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