

## BERNSTEIN FUNCTIONS OF SEVERAL SEMIGROUP GENERATORS ON BANACH SPACES UNDER BOUNDED PERTURBATIONS, II

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*Abstract.* The paper deals with multidimensional Bochner-Phillips functional calculus. In the previous paper by the author bounded perturbations of Bernstein functions of several commuting semigroup generators on Banach spaces were considered, conditions for Lipschitzness and estimates for the norm of commutators of such functions were proved. Also in the one-dimensional case the Fréchet differentiability of Bernstein functions of semigroup generators on Banach spaces were proved and a generalization of Livschits-Kreĭn trace formula derived. The aim of the present paper is to prove the Fréchet differentiability of operator Bernstein functions and the Livschits-Kreĭn trace formula in the multidimensional setting.

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*Keywords and phrases:* Bernstein function, Bochner-Phillips functional calculus, perturbation of an operator, Fréchet-differentiable function, semigroup of operators, spectral shift function, Livschits-Kreĭn trace formula, perturbation determinant.

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