

A NOTE ON THE RATIONAL CANONICAL FORM OF AN ENDOMORPHISM OF A VECTOR SPACE OF FINITE DIMENSION

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Abstract. In this note, we give an easy algorithm to construct the rational canonical form of a square matrix or an endomorphism h of a finite dimensional vector space which does not depend on either the structure theorem for finitely generated modules over principal ideal domains or matrices over the polynomial ring. The algorithm is based on the construction of an element whose minimum polynomial coincides with the minimum polynomial of the endomorphism and on the fact that the h -invariant subspace generated by such an element admits an h -invariant complement. It is also shown that this element can be easily obtained without the factorisation of a polynomial as a product of irreducible polynomials.

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