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EIGENVALUE SUMS OF COMBINATORIAL MAGNETIC LAPLACIANS ON FINITE GRAPHS

JOHN DEVER

Abstract. We give a construction of a class of magnetic Laplacian operators on finite directed graphs. We study some general combinatorial and algebraic properties of operators in this class before applying the Harrell-Stubbe Averaged Variational Principle to derive several sharp bounds on sums of eigenvalues of such operators. In particular, among other inequalities, we show that if G is a directed graph on n vertices arising from orienting a connected subgraph of d-regular loopless graph on n vertices, then if Δ_θ is any magnetic Laplacian on G, of which the standard combinatorial Laplacian is a special case, and $\lambda_0 \leqslant \lambda_1 \leqslant \ldots \leqslant \lambda_{n-1}$ are the eigenvalues of Δ_θ, then for $k \leqslant \frac{n}{2}$, we have

$$\frac{1}{k}\sum_{j=0}^{k-1}\lambda_j \leqslant d-1.$$

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