

QUADRATIC WEIGHTED GEOMETRIC MEAN IN HERMITIAN UNITAL BANACH $*$ -ALGEBRAS

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Abstract. In this paper we introduce the *quadratic weighted geometric mean*

$$x \mathbb{S}_v y := \left| |yx^{-1}|^v x \right|^2$$

for invertible elements x, y in a Hermitian unital Banach $*$ -algebra and real number v . We show that

$$x \mathbb{S}_v y = |x|^2 \sharp_v |y|^2,$$

where \sharp_v is the usual geometric mean and provide some inequalities for this mean under various assumptions for the elements involved.

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