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F_a–FRAME AND RIESZ SEQUENCES IN $L^2(\mathbb{R}_+)$

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Abstract. In application, $L^2(\mathbb{R}_+)$ can model casual signal space. This paper addresses the F_a-frame theory in $L^2(\mathbb{R}_+)$. The notion of F_a-frame for $L^2(\mathbb{R}_+)$ is somewhat like but distinct from that of frame. One of its special cases is a dilation-and-modulation frame for $L^2(\mathbb{R}_+)$. By intuition, F_a-frames have properties similar to usual frames. But they are nontrivial. In this paper, we introduce the notions of F_a-Bessel sequence and F_a-frame sequence in $L^2(\mathbb{R}_+)$. We characterize F_a-Bessel sequences, frame sequences and Riesz sequences, establish the links between F_a-Bessel sequences (F_a-frame sequences) and usual Bessel sequences (frame sequences), between F_a-orthonormal sequences and Parseval F_a-frames, and obtain an expansion with respect to Parseval F_a-frame sequences.

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