

## COMPLEMENTARITY OF SUBSPACES OF $\ell_\infty$ REVISITED

RYOTARO TANAKA

*Abstract.* We present a simple criterion for complementarity of subspaces of  $\ell_\infty$  induced by certain bounded linear operators. As applications, it is shown that some typical and well-known subspaces such as mean or almost convergent sequence spaces are uncomplemented in  $\ell_\infty$ . We also note that there exists a weak\* closed uncomplemented subspace of  $\ell_\infty$ .

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### REFERENCES

- [1] F. ALBIAC AND N. J. KALTON, *Topics in Banach space theory*, Springer, New York, 2006.
- [2] F. BAŞAR, *Summability theory and its applications*, Bentham Science Publishers, Ltd., Oak Park, IL, 2012.
- [3] F. BAŞAR AND M. KIRIŞÇI, *Almost convergence and generalized difference matrix*, *Comput. Math. Appl.* **61** (3) (2011), 602–611.
- [4] J. BOOS, *Classical and modern methods in summability*, Oxford University Press, Oxford, 2000.
- [5] R. M. DEVOS AND F. W. HARTMANN, *Sequences of bounded summability domains*, *Pacific J. Math.* **74** (2) (1978) 333–338.
- [6] S. FOUCAUT AND L. SKRZYPEK, *On maximal relative projection constants*, *J. Math. Anal. Appl.* **447** (1) (2017) 309–328.
- [7] J. D. HILL AND W. T. SLEDD, *Approximation in bounded summability fields*, *Canad. J. Math.* **20** (1968) 410–415.
- [8] H. KIZMAZ, *On certain sequence spaces*, *Canad. Math. Bull.* **24** (2) (1981) 169–176.
- [9] H. KÖNIG, *Spaces with large projection constants*, *Israel J. Math.* **50** (3) (1985) 181–188.
- [10] J. LINDENSTRAUSS, *On complemented subspaces of  $m$* , *Israel J. Math.* **5** (1967), 153–156.
- [11] G. G. LORENTZ, *A contribution to the theory of divergent sequences*, *Acta Math.* **80** (1948) 167–190.
- [12] R. E. MEGGINSON, *An Introduction to Banach Space Theory*, Springer-Verlag, New York, 1998.
- [13] M. NAKAMURA AND S. KAKUTANI, *Banach limits and the Čech compactification of a countable discrete set*, *Proc. Imp. Acad. Tokyo* **19** (1943) 224–229.
- [14] R. S. PHILLIPS, *On linear transformations*, *Trans. Amer. Math. Soc.* **48** (1940) 516–541.
- [15] M. ŞENGÖNÜ AND F. BAŞAR, *Some new Cesàro sequence spaces of non-absolute type which include the spaces  $c_0$  and  $c$* , *Soochow J. Math.* **31** (1) (2005) 107–119.
- [16] R. WHITLEY, *Mathematical Notes: Projecting  $m$  onto  $c_0$* , *Amer. Math. Monthly* **73** (3) (1966) 285–286.