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THE KUNZ–SOUILLARD APPROACH TO LOCALIZATION FOR JACOBI OPERATORS

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Abstract. In this paper we study spectral properties of Jacobi operators. In particular, we prove two main results: (1) that perturbing the diagonal coefficients of Jacobi operator, in an appropriate sense, results in exponential localization, and purely pure point spectrum with exponentially decaying eigenfunctions; and (2) we present examples of decaying *potentials* b_n such that the corresponding Jacobi operator has purely pure point spectrum.

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