

ON THE ESSENTIAL SPECTRA OF UNBOUNDED OPERATOR MATRICES WITH NON DIAGONAL DOMAIN AND AN APPLICATION

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Abstract. This paper is devoted to the investigation of the spectral stability of unbounded operator matrices with non diagonal domain in product of Banach spaces. Our results are aimed to characterize some essential spectra of this kind of operators in terms of the union of the essential spectra of the restriction of its diagonal operators entries. The abstract results are illustrated by an example of two-group transport equations with perfect periodic boundary conditions.

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