Operators
and
Matrices
Volume 13, Number 2 (2019), 349–361
doi:10.7153/oam-2019-13-25

# SPECTRAL MAPPING THEOREMS FOR WEYL SPECTRUM AND ISOLATED SPECTRAL POINTS

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*Abstract.* Spectral mapping theorems for Weyl spectrum and isolated spectral points were discussed by Gramsch, Lay and Oberai, etc. In this paper, $\mathscr{L}(\mathscr{X})$ means the space of all bounded linear operator on an infinite-dimensional complex Banach space $\mathscr{X}$, $f \in \mathscr{H}(\sigma(T))$ means $f$ is holomorphic on an open set $\mathscr{U}$ containing the spectrum $\sigma(T)$, and $f \in \mathscr{H}_{lnc}(\sigma(T))$ means $f$ is holomorphic and locally nonconstant. Firstly, it is shown that, if $T \in \mathscr{L}(\mathscr{X})$ and $f \in \mathscr{H}(\sigma(T))$, then (1) $\sigma_{uw}(f(T)) \subseteq f(\sigma_{uw}(T))$ where $\sigma_{uw}(T)$ means the upper semi-Weyl spectrum; (2) $\sigma_{uw}(f(T)) \supseteq f(\sigma_{uw}(T))$ is equivalent to the assertion that $T$ is of stable sign index on $\rho_{uf}(T)$ where $\rho_{uf}(T)$ means the upper semi-Fredholm resolvent. Secondly, let $T \in \mathscr{L}(\mathscr{X})$, (1) if $f \in \mathscr{H}_{lnc}(\sigma(T))$ or $T$ is polaroid, then $\sigma(f(T)) \backslash \pi_{00}(f(T)) \subseteq f(\sigma(T) \backslash \pi_{00}(T))$; (2) if $T$ is isoloid, then $\sigma(f(T)) \backslash \pi_{00}(f(T)) \supseteq f(\sigma(T) \backslash \pi_{00}(T))$. Some two-out-of-three results on spectral mapping theorems and Weyl type theorems are also given. At the end, an example is provided which implies that the conditions "$f \in \mathscr{H}_{lnc}(\sigma(T))$", "$T$ is polaroid" and "$T$ is isoloid" are crucial and inevitable.

*Mathematics subject classification* (2010): 47A53, 47A10.
*Keywords and phrases*: Weyl spectrum, B-Weyl spectrum, spectral mapping theorem, isolated spectral point, Weyl theorem.

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