

TRIANGULARIZABILITY OF FAMILIES OF POLYNOMIALLY COMPACT OPERATORS

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Abstract. A recent paper of Shemesh shows triangularizability of a pair $\{A, B\}$ of complex matrices satisfying the condition $A[A, B] = [A, B]B = 0$, or equivalently, the matrices A and B commute with their product AB . In this paper we extend this result to polynomially compact operators on Banach spaces. The case when the underlying space is Hilbert and one of operators is normal is also studied. Furthermore, we consider families of polynomially compact operators whose iterated commutators of some fixed length are zero. We also obtain a structure result in the special case of a finite family of algebraic operators.

Mathematics subject classification (2010): 47A15, 47A46, 47B07, 47B47, 16N20.

Keywords and phrases: Polynomially compact operators, triangularizability, Jacobson radical, commutator.

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