

## ON TWO EXTREMUM PROBLEMS RELATED TO THE NORM OF A BOUNDED LINEAR OPERATOR

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*Abstract.* We explore the norm attainment set and the minimum norm attainment set of a bounded linear operator between Hilbert spaces and Banach spaces. Indeed, we obtain a complete characterization of both the sets, separately for operators between Hilbert spaces and Banach spaces. We also study the interconnection between these two sets and prove that for operators between Hilbert spaces, these two sets are either equal or mutually orthogonal, provided both of them are non-empty. We also obtain separate complete characterizations of reflexive Banach spaces and Euclidean spaces in terms of the norm (minimum norm) attainment set, in order to illustrate the importance of our study.

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