

ORTHOMAPS ON FORMALLY REAL SIMPLE JORDAN ALGEBRAS

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Abstract. We characterize maps on finite-dimensional formally real simple Jordan algebras with the property $\phi(A \circ B) = \phi(A) \circ \phi(B)$ for all A, B . Although we do not assume additivity it turns out that every such map is either a real linear automorphism or a constant function. The main technique is a reduction to orthomaps, that is, maps which preserve zeros of Jordan product.

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