

## STRONGLY DRAZIN INVERSE IN RINGS

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*Abstract.* An element  $a \in R$  has strongly Drazin inverse if there exists  $a' \in R$  such that  $aa' = a'a, a' = a'aa'$  and  $a - aa' \in R$  is nilpotent, i.e.,  $a$  is strongly nil-clean. Additive results for strongly Drazin inverse in a ring are presented. The explicit formulas for such generalized inverse of  $a + b$  are given. These extend the results on s-Drazin and Drazin inverses of Wang (Filomat, **31**(2017), 1781–1789), Yang and Liu (J. Comput. Appl. Math., **235**(2011), 1412–1417). As an application, we give various conditions under which a  $2 \times 2$  block matrix has s-Drazin inverse.

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