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SPECTROGRAMS AND TIME–FREQUENCY LOCALIZED FUNCTIONS IN THE HANKEL SETTING

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Abstract. The uncertainty principle in Fourier analysis sets a limit to the possible simultaneous concentration of a function and its Hankel transform. Nevertheless, signals that have highly concentrated time–frequency content have many applications in quantum mechanics, PDE, engineering and in signal analysis. We use here time–frequency localization operators in the Hankel setting to measure the time–frequency content of functions on a subset of finite measure Σ within the time–frequency plane. Then, using eigenfunctions and eigenvalues of these operators, we prove a characterization of functions that are time–frequency concentrated in Σ, and we obtain approximation inequalities for such functions using a finite linear combination of eigenfunctions, since they are maximally time–frequency-concentrated in the region of interest.

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Paper OaM-13-39

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