

## A NEW BANACH SPACE DEFINED BY EULER TOTIENT MATRIX OPERATOR

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**Abstract.** The main object of this paper is to introduce a new Banach space derived by using a matrix operator which is comprised of Euler's totient function. Also, we determine  $\alpha$ ,  $\beta$ ,  $\gamma$ -duals of this space and characterize some matrix classes on this new space. Finally, we obtain necessary and sufficient conditions for some matrix operators to be compact.

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