

SELFADJOINT OPERATORS, NORMAL OPERATORS, AND CHARACTERIZATIONS

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Abstract. Let $\mathfrak{B}(H)$ be the C^* -algebra of all bounded linear operators acting on a complex separable Hilbert space H . We shall show that:

1. The class of all selfadjoint operators in $\mathfrak{B}(H)$ multiplied by scalars is characterized by

$$\forall X \in \mathfrak{B}(H), \|S^2X + XS^2\| \geq 2\|SXS\|, (S \in \mathfrak{B}(H)).$$

2. The class of all normal operators in $\mathfrak{B}(H)$ is characterized by each of the three following properties (where $D_S = S^*S - SS^*$, for $S \in \mathfrak{B}(H)$),

(i) $\forall X \in \mathfrak{B}(H), \|S^2X\| + \|XS^2\| \geq 2\|SXS\|, (S \in \mathfrak{B}(H)),$

(ii) $S^*D_S S = 0 = SD_S S^*, (S \in \mathfrak{B}(H)),$

(iii) $S^*D_S S \geq 0 \geq SD_S S^*, (S \in \mathfrak{B}(H)).$

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