

ISOLATED EIGENVALUES, POLES AND COMPACT PERTURBATIONS OF BANACH SPACE OPERATORS

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Abstract. Given a Banach space operator A , the isolated eigenvalues $E(A)$ and the poles $\Pi(A)$ (resp., eigenvalues $E^a(A)$ which are isolated points of the approximate point spectrum and the left poles $\Pi^a(A)$) of the spectrum of A satisfy $\Pi(A) \subseteq E(A)$ (resp., $\Pi^a(A) \subseteq E^a(A)$), and the reverse inclusion holds if and only if $E(A)$ (resp., $E^a(A)$) has empty intersection with the B-Weyl spectrum (resp., upper B-Weyl spectrum) of A . Evidently $\Pi(A) \subseteq E^a(A)$, but no such inclusion exists for $E(A)$ and $\Pi^a(A)$. The study of identities $E(A) = \Pi^a(A)$ and $E^a(A) = \Pi(A)$, and their stability under perturbation by commuting Riesz operators, has been of some interest in the recent past. This paper studies the stability of these identities under perturbation by (non-commuting) compact operators. Examples of analytic Toeplitz operators and operators satisfying the abstract shift condition are considered.

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