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CONSTANT NORMS AND NUMERICAL RADII OF MATRIX POWERS

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Abstract. For an n-by-n complex matrix A, we consider conditions on A for which the operator norms $\|A^k\|$ (resp., numerical radii $w(A^k)$), $k \geqslant 1$, of powers of A are constant. Among other results, we show that the existence of a unit vector x in $\mathbb{C}^n$ satisfying $|\langle A^k x, x\rangle| = w(A^k) = w(A)$ for $1 \leqslant k \leqslant 4$ is equivalent to the unitary similarity of A to a direct sum $\lambda B \oplus C$, where $|\lambda| = 1$, B is idempotent, and C satisfies $w(C^k) \leqslant w(B)$ for $1 \leqslant k \leqslant 4$. This is no longer the case for the norm: there is a 3-by-3 matrix A with $\|A^k x\| = \|A^k\| = \sqrt{2}$ for some unit vector x and for all $k \geqslant 1$, but without any nontrivial direct summand. Nor is it true for constant numerical radii without a common attaining vector. If A is invertible, then the constancy of $\|A^k\|$ (resp., $w(A^k)$) for $k = \pm 1, \pm 2, \ldots$ is equivalent to A being unitary. This is not true for invertible operators on an infinite-dimensional Hilbert space.

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