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ON THE MAXIMAL NUMERICAL RANGE OF A HYPONORMAL OPERATOR

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Abstract. Let A be a bounded linear operator acting on a complex Hilbert space. Let $\sigma(A)$ and $W_0(A)$ denote the spectrum and the maximal numerical range of A, respectively. In [10], it was shown that if A is a subnormal operator, then

$$W_0(A) = co(\{\lambda \in \sigma(A) : |\lambda| = \|A\|\}),$$

where $co(.)$ stands for the convex hull of the corresponding set. We extend this result to hyponormal operators. We give a geometric interpretation of the obtained result and deduce a necessary and sufficient condition to have $0 \in W_0(A)$ for a hyponormal operator A. Some properties of normaloid operators are also given.

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