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POSITIVE DEFINITENESS OF PIECEWISE–LINEAR FUNCTION 2

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Abstract. Let $\alpha,\beta \in (0,1)$, $0<\alpha \leqslant \beta < 1$, $s \in \mathbb{R}$ and let $w_{\alpha,\beta,s}$ be an even function with the properties: $w_{\alpha,\beta,s}(x)=0$ for $x>1$, $w_{\alpha,\beta,s}(0)=1$, $w_{\alpha,\beta,s}(1)=0$, $w_{\alpha,\beta,s}(x)=s$ for $x\in[\alpha,\beta]$ ($[\alpha,\alpha]:=\{\alpha\}$), $w_{\alpha,\beta,s}$ is linear over the intervals $[0,\alpha]$ and $[\beta,1]$. In this paper we prove that $w_{\alpha,\beta,s}$ is positive definite on $\mathbb{R} \iff m(\alpha,\beta)\leqslant s \leqslant M(\alpha,\beta)$, where $m(\alpha,\beta)\leqslant 0$, $M(\alpha,\beta)\geqslant 0$. If either $(1+\beta)/\alpha,(1-\beta)/\alpha \in \mathbb{N}$ or $1/\alpha \notin \mathbb{N}$, $\beta/\alpha \in \mathbb{N}$, then $M(\alpha,\beta)>0$, otherwise $M(\alpha,\beta)=0$. If either $(1+\beta)/\alpha,(1-\beta)/\alpha \in \mathbb{N}$ or $1/\alpha \in \mathbb{N}$, $\beta/\alpha \notin \mathbb{N}$, then $m(\alpha,\beta)<0$, otherwise $m(\alpha,\beta)=0$. Moreover, we find explicit values of $M(\alpha,\beta)$, $m(\alpha,\beta)$ for some α and β.

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