

INVERSE CONTINUITY OF THE NUMERICAL RANGE MAP FOR HILBERT SPACE OPERATORS

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Abstract. We describe continuity properties of the multivalued inverse of the numerical range map $f_A : x \mapsto \langle Ax, x \rangle$ associated with a linear operator A defined on a complex Hilbert space \mathcal{H} . We prove in particular that f_A^{-1} is strongly continuous at all points of the interior of the numerical range $W(A)$. We give examples where strong and weak continuity fail on the boundary and address special cases such as normal and compact operators.

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