

ON THE CLASS OF n -POWER D - m -QUASI-NORMAL OPERATORS ON HILBERT SPACES

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Abstract. As a continuation of our previous work [22], this paper is devoted to the study for further properties of the class of (n, m) -power D -normal operators $([(n, m)DN])$ and introduce some classes of operators on Hilbert space called D - m -quasi-normal operators and it is denoted by $([D(QN)^m])$, n -power D - m -quasi-normal operators and it is denoted by $([nD(QN)^m])$, associated with a Drazin invertible operator using its Drazin inverse. Some characterizations of D - m -quasi-normal and n -power D - m -quasi-normal operators are discussed. Inclusion relations among the various classes of normal operators are characterized.

Mathematics subject classification (2010): 47B15, 47B20, 47A15.

Keywords and phrases: (n, m) -power normal, (n, m) -power quasi-normal, n -power D -normal, n -power D -quasi-normal.

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