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LINEAR MAPS ON BLOCK UPPER TRIANGULAR MATRIX ALGEBRAS BEHAVING LIKE JORDAN DERIVATIONS THROUGH COMMUTATIVE ZERO PRODUCTS

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Abstract. Let $\mathscr{T} = \mathscr{T}(n_1, n_2, \cdots, n_k) \subseteq M_n(\mathscr{C})$ be a block upper triangular matrix algebra and let $\mathscr{M}$ be a 2-torsion free unital $\mathscr{T}$-bimodule, where $\mathscr{C}$ is a commutative ring. Let $\Delta : \mathscr{T} \to \mathscr{M}$ be a $\mathscr{C}$-linear map. We show that if $\Delta(X)Y + X\Delta(Y) + \Delta(Y)X + Y\Delta(X) = 0$ whenever $X, Y \in \mathscr{T}$ are such that $XY = YX = 0$, then $\Delta(X) = D(X) + \alpha(X) + X\Delta(I)$, where $D : \mathscr{T} \to \mathscr{M}$ is a derivation, $\alpha : \mathscr{T} \to \mathscr{M}$ is an antiderivation, I is the identity matrix and $\Delta(I)X = X\Delta(I)$ for all $X \in \mathscr{T}$. We also prove that under some sufficient conditions on $\mathscr{T}$, we have $\alpha = 0$. As a corollary, we show that under given sufficient conditions, each Jordan derivation $\Delta : \mathscr{T} \to \mathscr{M}$ is a derivation and this is an answer to the question raised in [9]. Some previous results are also generalized by our conclusions.

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