

NORMS, KERNELS AND EIGENVALUES OF SOME INFINITE GRAPHS

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Abstract. In this paper we study the adjacency matrix of some infinite graphs, which we call the shift operator on the L^p space of the graph. In particular, we establish norm estimates, we find the norm for some cases, we decide the triviality of the kernel of some infinite trees, and we find the eigenvalues of certain infinite graphs obtained by attaching an infinite tail to some finite graphs.

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