

A NOTE ON A CONJECTURED SINGULAR VALUE INEQUALITY RELATED TO A LINEAR MAP

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Abstract. If $\begin{pmatrix} A & D \\ D^* & C \end{pmatrix}$ is positive semidefinite with each block $n \times n$, Lin conjectured that

$$s_j(\Phi(D)) \leq s_j(\Phi(A)\sharp\Phi(C)), \quad j = 1, \dots, n,$$

where Φ is the linear map: $D \mapsto D + (\operatorname{tr}D)I_n$ and $s_j(D)$ denotes the j -th largest singular value of the matrix D . In this note, we confirm this conjecture when $n = 2$.

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