

ON THE BOUNDEDNESS OF SUBSEQUENCES OF VILENKIN-FEJÉR MEANS ON THE MARTINGALE HARDY SPACES

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Abstract. In this paper we characterize subsequences of Fejér means with respect to Vilenkin systems, which are bounded from the Hardy space H_p to the Lebesgue space L_p , for all $0 < p < 1/2$. The result is in a sense sharp.

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