

PARSEVAL FRAMES OF PIECEWISE CONSTANT FUNCTIONS

DORIN ERVIN DUTKAY AND RAJITHA RANASINGHE

Abstract. We present a way to construct Parseval frames of piecewise constant functions for $L^2[0, 1]$. The construction is similar to the generalized Walsh bases. It is based on iteration of operators that satisfy a Cuntz-type relation, but without the isometry property. We also show how the Parseval frame can be dilated to an orthonormal basis and the operators can be dilated to true Cuntz isometries.

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