Operators
and
Matrices
Volume 14, Number 2 (2020), 333–349
doi:10.7153/oam-2020-14-26

ON THE LOCAL SPECTRAL PROPERTIES OF THE LEFT MULTIPLICATION OPERATORS

ZINE EL ABIDINE ABDELALI

Abstract. Let X and Y be complex Banach spaces, and $B(Y,X)$ (resp. $B(X)$) be the space of all bounded linear operators from Y into X (resp. from X into itself). Fix an operator $T\in B(X)$ and an open subset U of $\mathbb{C}$, and denote by L_T the left multiplication operator on $B(Y,X)$ induced by T. Let

$$\mathscr{X}_T(\mathbb{C}\setminus U):=\{x\in X:(T-\lambda)f(\lambda)=x \text{ has an analytic solution } f \text{ on } U\}$$

denote the glocal spectral subspace of T on $\mathbb{C}\setminus U$. In this paper, we establish an operator valued factorization theorem type when X is a Hilbert space or Y is an ℓ^1-space, and prove that

$$\left\{Q\in B(Y,X):Q(Y)\subseteq\mathscr{X}_T(\mathbb{C}\setminus U)\right\}\subseteq\mathscr{B}(\mathscr{Y},\mathscr{X})_{L_T}(\mathbb{C}\setminus O)$$

for all nonempty relatively compact open subsets O of U. We also prove that if T has the single-valued extension property (SVEP), then

$$\mathscr{B}(\mathscr{Y},\mathscr{X})_{L_T}(\mathbb{C}\setminus U)=\left\{Q\in B(Y,X):Q(Y)\subseteq\mathscr{X}_T(\mathbb{C}\setminus U)\right\}.$$

Furthermore, we characterize the local spectra and the glocal spectral spaces of the left multiplication operators on $B(\mathscr{A})$ where $\mathscr{A}$ stands for certain Banach spaces and algebras. Moreover, we introduce and study some natural extensions of local, surjective and right spectra of any operator $S\in B(X)$, mainly the minimal and maximal local spectra of S at paracomplete subspaces of X.

Mathematics subject classification (2010): 47A11, 47B99, 46A22.
Keywords and phrases: Local spectrum, vector valued analytic functions, single valued extension property, glocal spectral space, local spectral space, ℓ^1-spaces, paracomplete spaces.

REFERENCES

[1] Z. ABDELALI AND A. BOURHIM, *Maps preserving the local spectrum of quadratic products of matrices*, Acta Sci. Math. (Szeged) **84**, no. 1-2, (2018) 49–64.
[2] Z. ABDELALI, A. BOURHIM AND M. MABROUK, *Lie product and local spectrum preservers*, Linear Algebra Appl. **553** (2018) 328–361.
[3] Z. ABDELALI, A. ACHCHI AND R. MARZOUKI, *Maps preserving the local spectrum of some matrix products*, Oper. Matrices **12**, no. 2, (2018) 549–562.
[4] Z. ABDELALI, A. ACHCHI AND R. MARZOUKI, *Maps preserving the local spectrum of skew-product of operators*, Linear Algebra Appl. **485** (2015) 58–71.
[5] A. ACHCHI, M. MABROUK AND R. MARZOUKI, *Maps preserving the local spectrum of the skew Jordan product of operators*, Oper. Matrices **11**, no. 1, (2017) 133–146.
[6] P. AIENA, *Fredholm and Local Spectral Theory, with Applications to Multipliers*, Kluwer, (2004).
[7] G.R. ALLAN, *On one-sided inverses in Banach algebras of holomorphic vectorvalued functions*, J. London Math. Soc., **42** (1967), 463–470.
[8] N. BOURBAKI, *Espaces Vectoriels Topologiques*, Chap.1-5, Masson (1981).
[9] A. BOURHIM, M. BURGOS AND V.S. SHULMAN, *Linear maps preserving the minimum and reduced minimum moduli*, J. Funct. Anal., **258** (2010), 50–66.

© ELEMENT, Zagreb
Paper OaM-14-26

[10] A. BOURHIM AND J. MASHREGHI, *Local spectral radius preservers*, Integral Equations Operator Theory, **76**(1) (2013), 95–104.
[11] A. BOURHIM AND J. MASHREGHI, *A survey on preservers of spectra and local spectra, in: Invariant Subspaces of the Shift operator*, Contemp. Math., **638**, Amer. Math. Soc, Providence, RI (2015), 45–98.
[12] A. BOURHIM AND T. RANSFORD, *Additive maps preserving local spectrum*, Integral Equations Operator Theory, **55** (2006), 377–385.
[13] J. BRAČIČ AND V. MÜLLER, *Local spectrum and local spectral radius of an operator at a fixed vector*, Studia Math., **194**(2) (2009), 155–162.
[14] M. BREŠAR, M.A. CHEBOTAR AND W.S. MARTINDALE 3RD, *Functional Identities*, Schwitzerland: Birkhäuser Verlag (2007).
[15] N.L. CAROTHERS, *A Short Course on Banach Space Theory*, London Mathematical Society, Student Texts 64. Cambridge University Press (2005).
[16] R.W. CROSS, *On the continuous linear image of a Banach space*, J. Austral. Math. Soc. A **29** (1980), 219–234.
[17] H.G. DALES, *Banach Algebras and Automatic Continuity*, London Math. Soc. Monographs, New Series 24, Oxford Science Publications, The Clarence Press, Oxford University Press, New York (2000).
[18] J. DIESTEL, *Sequences and Series in Banach Spaces*, Graduate Texts in Mathematics **92**, Springer-Verlag New York Inc. (1984).
[19] J. DIESTEL AND J. J. UHL, *Vector measures*, Mathematical Surveys, No. 15. American Mathematical Society, Providence, Rhode Island (1977).
[20] N. DUNFORD AND J.T. SCHWARTZ, *Linear operators I*, Wiley-Interscience, New York (1958).
[21] J. ESCHMEIER AND M. PUTINAR, *Spectral Decompositions and Analytic Sheaves*, London Math. Soc. Monographs New Series **10**, Oxford Univ. Press (1996).
[22] J. ESCHMEIER, K.B. LAURSEN, AND M.M. NEUMANN, *Multipliers with natural local spectra on commutative Banach algebras*, J. Funct. Anal., **138** (1996), 273–294.
[23] R. HARTE, *Invertibility and Singularity for Bounded Linear Operators*, Marcel Dekker, Inc., New York and Basel (1988).
[24] R. HARTE, *On local spectral theory II*, Functional Analysis, Approximation and Computation 2:1 (2010), 67–71.
[25] G. KÖTHE, *Hebbare localkonvexe räume*, Math. Ann., **165** (1966), 181–195.
[26] K. B. LAURSEN AND M. M. NEUMANN, *An Introduction to Local Spectral Theory*, London Math. Soc. Monographs New Series, Clarendon Press, Oxford (2000).
[27] J. LEITERER, *Banach coherent analytic Fréchet sheaves*, Math. Nachr., **85** (1978), 91–109.
[28] J. LINDENSTRAUSS AND L. TZAFRIRI, *Classecal Banach Spaces I*, Sequence Spaces, Springer-Verlag (1977).
[29] T.L. MILLER, V.G. MILLER, M.M. NEUMANN, *Local spectral properties of weighted shifts*, J. Operator Theory, **51** (2004), 71–88.
[30] V. MÜLLER, *Spectral Theory of Linear Operators and Spectral Systems in Banach Algebras*, Operator Theory: Advances and Applications **139**, Birkhäuser Verlag, Basel (2003).
[31] G.J. MURPHY, *C^*-algebras and operator theory*, Academic Press (1990).
[32] A. PEŁCZYŃSKY, *Projections in certain Banach spaces*, Studia Math. **19** (1960), 209–228.
[33] A. PEŁCZYŃSKY AND C. BESSAGA, *Some aspects of the present theory of Banach spaces, in "S. Banach, Oeuvres", **II***, PWN Warszawa (1979), 221–302.
[34] W. RUDIN, *Functional analysis (2nd edn)*, New York, McGraw-Hill (1991).