

AN INDEFINITE RANGE INCLUSION THEOREM FOR TRIPLETS OF BOUNDED LINEAR OPERATORS ON A HILBERT SPACE

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Abstract. We study triplets of Hilbert space operators satisfying a certain inequality. A range inclusion theorem with norm estimate for those triplets is given with the language of Kreĭn space geometry and de Branges-Rovnyak space theory.

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