

## ON THE $k$ POINT DENSITY PROBLEM FOR BAND-DIAGONAL $M$ -BASES

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*Abstract.* In the early 1990s the works of Larson, Wogen and Argyros, Lambrou, Longstaff disclosed an example of a strong tridiagonal  $M$ -basis that was not rank one dense. Later Katavolos, Lambrou and Papadakis studied  $k$  point density property of this example. In this paper we present new methods for the analysis of  $k$  point density and rank one density properties for band-diagonal  $M$ -bases.

*Mathematics subject classification (2010):* 46A35.

*Keywords and phrases:* Biorthogonal system,  $M$ -basis, rank one density, two point density.

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